User cost of credit card services under intertemporal nonseparability

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Research question

- What is the risk adjustment for the user cost of credit card services?
  1. Credit card transactions are important elements in monetary aggregate.
  2. Interest rates on credit cards are high and volatile; therefore, the risk adjustment on credit card user cost can be high.

Literature review

- Barnett and Su (2016) first developed the user cost and risk adjustment of credit card services.
- The risk adjustment they found is likely to be small under the assumption of intertemporal separability in consumption.

Contribution

- We introduce intertemporal nonseparability in consumption.
- We are expecting to produce more accurate and larger risk adjustment on credit card user cost.

Consumer's optimization problem

$$\mathrm{E}_t \sum_{s=0}^{\infty} \beta^s U(\mathbf{m}_{t+s}, x_{t+s}, x_{t+s-1}, \ldots, x_{t+s-n}), \quad (1)$$

subject to the following budget constraints,

$$W_t = p_t^* x_t + p_t^* k_t + p_t^* \mathbf{m}_t^a - p_t^* \mathbf{m}_t^c - p_t^* \mathbf{z}_t \quad (2)$$

and

$$W_{t+1} = (\mathbb{1} + \mathbf{r}_{t+1}) p_t^* \mathbf{m}_t^a - (\mathbb{1} + \mathbf{e}_{t+1}) p_t^* \mathbf{m}_t^c - (\mathbb{1} + \mathbf{e}_{t+1}^c) p_t^* \mathbf{z}_t + (1 + R_{t+1} p_t^*) k_t + y_{t+1} \quad (3)$$

The consumer also is subject to the transversality condition,

$$\lim_{s \to \infty} \beta^s p_t^* k_{t+s} = 0 \quad (4)$$

Euler equations

$$\mathrm{E}_t\left(\frac{\partial U_t}{\partial \mathbf{m}_t^a} + \beta\lambda_{t+1}\frac{p_t^*}{p_{t+1}^*}(\mathbb{1} + \mathbf{r}_{t+1}) - \lambda_t\mathbb{1}\right) = 0 \tag{5}$$

$$\mathrm{E}_t\left(\frac{\partial U_t}{\partial \mathbf{m}_t^c} - \beta\lambda_{t+1}\frac{p_t^*}{p_{t+1}^*}(\mathbb{1} + \mathbf{e}_{t+1}) + \lambda_t\mathbb{1}\right) = 0 \tag{6}$$

$$\mathrm{E}_t\left(\beta\lambda_{t+1}\frac{p_t^*}{p_{t+1}^*}(1 + R_{t+1}) - \lambda_t\right) = 0 \tag{7}$$

Define $\lambda_t = \mathrm{E}_t \sum_{i=0}^{n} \beta^i \frac{\partial U_{t+i}}{\partial x_t}$, where λ_t is the expected present value of the marginal utility of consumption x_t.

When the utility function is intertemporally separable, $\lambda_t = \frac{\partial U_t}{\partial x_t}$.
Rearranging equation

$$E_t\left(\frac{\partial U_t}{\partial m^a_{i,t}} - \beta\frac{p^*_t}{p^*_{t+1}}(R_t - r_{i,t})\frac{\partial U_t}{\partial x_{t+1}}\right) = 0$$
$$E_t\left(\frac{\partial U_t}{\partial m^c_{l,t}} - \beta\frac{p^*_t}{p^*_{t+1}}(e_{l,t} - R_t)\frac{\partial U_t}{\partial x_{t+1}}\right) = 0$$
$$E_t\left(\frac{\partial U_t}{\partial x_t} - \beta\frac{p^*_t}{p^*_{t+1}}(1 + R_{t+1})\frac{\partial U_t}{\partial x_{t+1}}\right) = 0 \tag{8}$$

We can see that the general result in Barnett and Su (2016) still holds. It is a special case under intertemporal separability.

Define the contemporaneous real user-cost price of the services of **monetary asset** i

$$\pi_{i,t}^{a} = \frac{\frac{\partial U_t}{\partial \mathbf{m}_t^a}}{E_t\left(\frac{\partial U_t}{\partial x_t} + \beta \frac{\partial U_{t+1}}{\partial x_t} + \cdots + \beta^n \frac{\partial U_{t+n}}{\partial x_t}\right)} = \frac{\frac{\partial U_t}{\partial \mathbf{m}_t^a}}{\lambda_t} \quad (9)$$

Define the contemporaneous real user-cost price of the services of **credit card transaction services** l

$$\pi_{l,t}^{c} = \frac{\frac{\partial U_t}{\partial \mathbf{m}_t^c}}{E_t\left(\frac{\partial U_t}{\partial x_t} + \beta \frac{\partial U_{t+1}}{\partial x_t} + \cdots + \beta^n \frac{\partial U_{t+n}}{\partial x_t}\right)} = \frac{\frac{\partial U_t}{\partial \mathbf{m}_t^c}}{\lambda_t} \quad (10)$$

Proposition

Let $s_{i,t} = \pi^a_{i,t} m^a_{i,t} / \sum_{i=1}^{I} \pi^a_{i,t} m^a_{i,t}$ *be the user-cost-evaluated expenditure share of monetary assets and* $s_{l,t} = \pi^c_{l,t} m^c_{l,t} / \sum_{l=1}^{L} \pi^c_{l,t} m^c_{l,t}$ *be the user-cost-evaluated expenditure share of credit card transactions. Under the weak-separability assumption, we have for any linearly homogenous monetary aggregator function,* $M(.)$, *that*

$$\mathrm{d} \log M_t = \mathbf{s}_t \mathrm{d} \log \mathbf{m}_t \tag{11}$$

which can also be written as

$$\mathrm{d} \log M_t = \sum_{i=1}^{I} s_{i,t} \mathrm{d} \log m^a_{i,t} + \sum_{l=1}^{L} s_{l,t} \mathrm{d} \log m^c_{l,t} \tag{12}$$

where $M_t = M(\mathbf{m}_t)$.

We define the pricing kernel to be

$$Q_{t+1} = \beta\lambda_{t+1}/\lambda_t \tag{13}$$

λ_t is the present value of the marginal utility of consumption at time t. Q_{t+1} is the subjectively-discounted marginal rate of substitution between consumption. If the utility function is time-separable, we have that $Q_{t+1} = \beta\frac{\partial U(\mathbf{m}_{t+1}, x_{t+1})/\partial x_{t+1}}{\partial U(\mathbf{m}_t, x_t)/\partial x_t}$.

How does pricing kernel help approximate risk adjustment?

- It reflects the trade-off among monetary services, risk, and rate of return on different assets through the first-order conditions.
- This is similar to the Divisia index, which tracks the monetary aggregate according to the rate of return on different monetary aggregates through the first-order conditions.

$$0 = 1 - \mathrm{E}_t(\tilde{r}_{j,t+1} Q_{t+1}) \tag{14}$$

$$\pi^a_{i,t} = 1 - \mathrm{E}_t(r_{i,t+1} Q_{t+1}) \tag{15}$$

$$\pi^c_{l,t} = \mathrm{E}_t(e_{l,t+1} Q_{t+1}) - 1 \tag{16}$$

Interpretations

- For monetary assets, equation 15 implies that the "deviation" from the usual Euler equation measures the user cost of that monetary asset.
- For credit card transactions, equation 16 implies that "deviation" from the usual Euler equation measures the user cost of the credit card transaction.

Proposition

Given the real rate of return, $r_{i,t+1}$, on a monetary asset and the real rate of return, $\tilde{r}_{j,t+1}$, on a non-monetary asset, and $e_{l,t+1}$ the real interest rate on a credit card transaction, the risk-adjusted real user-cost price of the services of the monetary asset can be obtained as

$$\pi^a_{i,t} = \frac{(1+\omega_{i,t})E_t\tilde{r}_{j,t+1} - (1+\omega_{j,t})\,E_t\,r_{i,t+1}}{E_t\,\tilde{r}_{j,t+1}} \quad (17)$$

$$\omega_{i,t} = -\,\mathrm{Cov}_t(Q_{t+1}, r_{i,t+1}) \quad (18)$$

$$\omega_{j,t} = -\,\mathrm{Cov}_t(Q_{t+1}, \tilde{r}_{j,t+1}) \quad (19)$$

Proposition

The risk-adjusted real user-cost price of the services of the credit card transactions can be obtained as

$$\pi^c_{l,t} = \frac{(1+\omega_{l,t})E_t\tilde{r}_{j,t+1} - (1+\omega_{j,t})E_t e_{l,t+1}}{\mathrm{E}_t\,\tilde{r}_{j,t+1}} \quad (20)$$

$$\omega_{l,t} = -\mathrm{Cov}_t(Q_{t+1}, e_{l,t+1}) \quad (21)$$

$$\omega_{j,t} = -\mathrm{Cov}_t(Q_{t+1}, \tilde{r}_{j,t+1}) \quad (22)$$

Under uncertainty we can choose any non-monetary asset as the ”benchmark” asset, when computing the risk-adjusted user-cost prices of the services of monetary assets.

- We extend the monetary-asset user cost to the case of multiple non-monetary assets.
- The proposition relates the user cost of a monetary asset to the rates of return on financial assets, which need not be risk free.
- It applies to an arbitrary pair of monetary and non-monetary assets.

$$\pi^e_{i,t} = \frac{r^f_t - \mathrm{E}_t\, r_{i,t+1}}{r^f_t} \tag{23}$$

The first-order condition for r^f_t is

$$1 = \mathrm{E}_t(Q_{t+1} r^f_t) \tag{24}$$

Hence, we have, from the nonrandomness of r^f_t, that

$$\mathrm{E}_t(Q_{t+1}) = \frac{1}{r^f_t} \tag{25}$$

Replacing $\mathrm{E}_t(Q_{t+1})$ in equation 15, we then have

$$\pi^a_{i,t} = \frac{r^f_t - \mathrm{E}_t\, r_{i,t+1}}{r^f_t} + \omega_{i,t} = \pi^e_{i,t} + \omega_{i,t} \tag{26}$$

where $\omega_{i,t} = -\mathrm{Cov}_t(Q_{t+1}, r_{i,t+1})$. Similarly, we have,

$$\pi^c_{l,t} = \frac{\mathrm{E}_t\, e_{l,t+1} - r^f_t}{r^f_t} - \omega_{l,t} = \pi^e_{l,t} - \omega_{l,t} \tag{27}$$

where $\omega_{l,t} = -\mathrm{Cov}_t(Q_{t+1}, e_{l,t+1})$.

Simple sum index:

$$\sum_{i=1}^{I} m_{i,t}^{a} = \sum_{i=1}^{I} \frac{r_t^f - Er_{i,t}}{r_t^f} m_{i,t}^{a} + \omega_{i,t} + \sum_{i=1}^{I} \frac{Er_{i,t}}{r_t^f} m_{i,t}^{a} - \omega_{i,t} \tag{28}$$

$\sum_{i=1}^{I} \frac{r_t^f - Er_{i,t}^a}{r_t^f} m_{i,t}^{a} + \omega_{i,t}$ is the discounted monetary service flow part.
$\sum_{i=1}^{I} \frac{Er_{i,t}}{r_t^f} m_{i,t}^{a}$ is the discounted investment yield.
$\omega_{i,t}$ is the risk adjustment.
Under the assumption that all monetary assets yield zero interest and people are risk neutral. It follows that:

$$\sum_{i=1}^{I} \frac{r_t^f - Er_{i,t}^a}{r_t^f} m_{i,t}^{a} + \omega_{i,t} = \sum_{i=1}^{I} m_{i,t}^{a} \tag{29}$$

If the utility function is time-separable, $Q_{t+1} = \beta \frac{\partial U(\mathbf{m}_{t+1}, x_{t+1})/\partial x_{t+1}}{\partial U(\mathbf{m}_t, x_t)/\partial x_t}$.

$$\begin{aligned} \pi_{i,t}^{a} = & \frac{\mathrm{E}_t\, \tilde{r}_{j,t+1} - \mathrm{E}_t\, r_{i,t+1}}{\mathrm{E}_t\, \tilde{r}_{j,t+1}} \\ & + \frac{-\beta\, \mathrm{Cov}_t(\partial U(\mathbf{m}_{t+1}, x_{t+1})/\partial x_{t+1}, r_{i,t+1})}{\partial U(\mathbf{m}_t, x_t)/\partial x_t} \\ & + \frac{\beta(1-\pi_{i,t}^{e})\, \mathrm{Cov}_t(\partial U(\mathbf{m}_{t+1}, x_{t+1})/\partial x_{t+1}, \tilde{r}_{j,t+1})}{\partial U(\mathbf{m}_t, x_t)/\partial x_t} \end{aligned} \tag{30}$$

We can see that the user cost of monetary assets found in Barnett and Su (2016) is a special case under intertemporal separability.

Similarly, we have

$$\pi^c_{l,t} = \frac{\mathrm{E}_t\, \tilde{r}_{j,t+1} - \mathrm{E}_t\, e_{l,t+1}}{\mathrm{E}_t\, \tilde{r}_{j,t+1}} + \frac{-\beta\, \mathrm{Cov}_t(\partial U(\mathbf{m}_{t+1}, x_{t+1})/\partial x_{t+1}, e_{l,t+1})}{\partial U(\mathbf{m}_t, x_t)/\partial x_t} + \frac{\beta(1-\pi^c_{l,t})\, \mathrm{Cov}_t(\partial U(\mathbf{m}_{t+1}, x_{t+1})/\partial x_{t+1}, \tilde{r}_{j,t+1})}{\partial U(\mathbf{m}_t, x_t)/\partial x_t} \tag{31}$$

where the first part of the equation represents the risk-free user cost of credit card services, the rest represents the risk adjustment. We can see that the user cost of monetary assets found in Barnett and Su (2016) is a special case under intertemporal separability.

Approximation to the risk adjustment

- In CCAPM, beta is used to measure the risk adjustment. Asset i's risk premium depends on its market portfolio risk exposure, which is measured by the beta of this asset. $\mathrm{E}_t\, r_{M,t+1} - r_t^f$, in accordance with

$$\mathrm{E}_t\, r_{i,t+1} - r_t^f = \beta_{i,t}(\mathrm{E}_t\, r_{A,t+1} - r_t^f) \tag{32}$$

  where $\beta_{i,t} = \mathrm{Cov}_t(r_{i,t+1} - r_t^f, r_{A,t+1} - r_t^f)/\ \mathrm{Var}_t(r_{A,t+1} - r_t^f)$.
- We will introduce beta to measure the risk adjustment in credit card services user cost.

Let A_t be the portfolio's stock value, $\phi_{i,t}$, $\varphi_{j,t}$ and $\psi_{l,t}$ denote the share of $m^a_{i,t}$, $k_{j,t}$ and $m^c_{l,t}$ in the portfolio's stock value, respectively, so that

$$\phi_{i,t} = \frac{m^a_{i,t}}{\sum_{i=1}^{I} m^a_{i,t} + \sum_{j=1}^{J} k_{j,t} + \sum_{l=1}^{L} m^c_{l,t}} = \frac{m^a_{i,t}}{A_t} \quad (33)$$

$$\varphi_{j,t} = \frac{k_{j,t}}{\sum_{i=1}^{I} m^a_{i,t} + \sum_{j=1}^{J} k_{j,t} + \sum_{l=1}^{L} m^c_{l,t}} = \frac{k_{j,t}}{A_t} \quad (34)$$

$$\psi_{l,t} = \frac{m^c_{l,t}}{\sum_{i=1}^{I} m^a_{i,t} + \sum_{j=1}^{J} k_{j,t} + \sum_{l=1}^{L} m^c_{l,t}} = \frac{m^c_{l,t}}{A_t} \quad (35)$$

Then, by construction,

$$r_{A,t+1} = \sum_{i=1}^{I} \phi_{i,t} r_{i,t+1} + \sum_{j=1}^{J} \varphi_{j,t} \tilde{r}_{j,t+1} + \sum_{l=1}^{L} \psi_{l,t} e_{l,t+1} \tag{36}$$

Multiplying 14 by $\phi_{i,t}$, multiplying 15 by $\psi_{l,t}$, multiplying 16 by $\varphi_{j,t}$, we have

$$0 = \varphi_{j,t} - \mathrm{E}_t(Q_{t+1}\varphi_{j,t}\tilde{r}_{j,t+1}) \tag{37}$$

$$\phi_{i,t}\pi^a_{i,t} = \phi_{i,t} - \mathrm{E}_t(Q_{t+1}\phi_{i,t}r_{i,t+1}) \tag{38}$$

$$\psi_{l,t}\pi^c_{l,t} = \mathrm{E}_t(Q_{t+1}\psi_{l,t}e_{l,t+1}) - \psi_{l,t} \tag{39}$$

Summing over equations 36, 37 and minus equation 38, and using the definition of $r_{A,t+1}$, we get

$$\sum_{i=1}^{I} \phi_{i,t}\pi^a_{i,t} - \sum_{l=1}^{L} \psi_{l,t}\pi^c_{l,t} = 1 - \mathrm{E}_t(Q_{t+1}r_{A,t+1}) \tag{40}$$

Let $\Pi_{A,t} = \sum_{i=1}^{I} \phi_{i,t}\pi^a_{i,t} + \sum_{j=1}^{J} \varphi_{j,t}\tilde{\pi}_{j,t} - \sum_{l=1}^{L} \psi_{l,t}\pi^c_{l,t}$.

We can show that our definition of $\Pi_{A,t}$ is consistent with Fisher's factor reversal test, as follows:

Proposition

The pair $(A_t, \Pi_{A,t})$ satisfies factor reversal, defined by:

$$\Pi_{A,t} A_t = \sum_{i=1}^{I} \pi_{i,t}^{a} m_{i,t}^{a} + \sum_{j=1}^{J} \tilde{\pi}_{j,t} k_{j,t} - \sum_{l=1}^{L} \pi_{l,t}^{c} m_{l,t}^{c} \tag{41}$$

Since we know that $\tilde{\pi}_{j,t} = 0$ for all j, factor reversal equivalently can be written as

$$\Pi_{A,t} A_t = \sum_{i=1}^{I} \pi_{i,t}^{a} m_{i,t}^{a} - \sum_{l=1}^{L} \pi_{l,t}^{c} m_{l,t}^{c} \tag{42}$$

Proposition

If one of the non-monetary assets is (locally) risk-free with gross real interest rate r_t^f, and if $Q_{t+1} = a_t - b_t r_{A,t+1}$, where $r_{A,t+1}$ is the gross real rate of return on the consumer's wealth portfolio, then the user cost of any monetary asset i is given by

$$\pi_{i,t}^a - \pi_{i,t}^e = \beta_{i,t}(\Pi_{A,t} - \Pi_{A,t}^e) \tag{43}$$

$$\pi_{l,t}^c - \pi_{l,t}^e = \beta_{l,t}(\Pi_{A,t} - \Pi_{A,t}^e) \tag{44}$$

where $\pi_{i,t}^a$, $\pi_{l,t}^c$, and $\Pi_{A,t}$ are the user costs of monetary asset i, user costs of credit card transaction l and user costs of the asset wealth portfolio, respectively, and $\pi_{i,t}^e = \frac{r_t^f - \mathrm{E}_t\, r_{i,t+1}}{r_t^f}$, $\pi_{l,t}^e = \frac{\mathrm{E}_t\, e_{l,t+1} - r_t^f}{r_t^f}$, and $\Pi_{A,t}^e = \frac{r_t^f - \mathrm{E}_t\, r_{A,t+1}}{r_t^f}$.

$$\pi^a_{i,t} - \pi^e_{i,t} = \beta_{i,t}(\Pi_{A,t} - \Pi^e_{A,t}) \tag{45}$$
$$\pi^c_{l,t} - \pi^e_{l,t} = \beta_{l,t}(\Pi_{A,t} - \Pi^e_{A,t}) \tag{46}$$

where

$$\beta_{i,t} = \frac{\mathrm{Cov}_t(r_{A,t+1}, r_{i,t+1})}{\mathrm{Var}_t(r_{A,t+1})} \tag{47}$$
$$\beta_{l,t} = \frac{\mathrm{Cov}_t(-r_{A,t+1}, e_{l,t+1})}{\mathrm{Var}_t(r_{A,t+1})} \tag{48}$$

Using the assumption that $Q_{t+1} = a_t - b_t r_{A,t+1}$, so that $\text{Cov}_t(Q_{t+1}, r_{A,t+1}) = -b_t \text{Var}_t(r_{A,t+1})$, it follows that

$$\begin{aligned} \pi^a_{i,t} &= \pi^e_{i,t} - \text{Cov}_t(Q_{t+1}, r_{i,t+1}) \\ &= \pi^e_{i,t} + b_t \text{Cov}_t(r_{A,t+1}, r_{i,t+1}) \end{aligned} \tag{49}$$

$$\begin{aligned} \pi^c_{l,t} &= \pi^e_{l,t} + \text{Cov}_t(Q_{t+1}, e_{l,t+1}) \\ &= \pi^e_{l,t} - b_t \text{Cov}_t(r_{A,t+1}, e_{l,t+1}) \\ &= \pi^e_{l,t} + b_t \text{Cov}_t(-r_{A,t+1}, e_{l,t+1}) \end{aligned} \tag{50}$$

- The results implies that asset i's risk premium depends on its market portfolio risk exposure, which is measured by the beta of this asset.
- Our proposition shows that the risk adjustment to the certainty equivalent user cost of asset i is determined in that manner as well.
- The larger the risk exposure to the wealth portfolio, the larger the risk adjustment.

Quadratic utility form

Assumption

Let V have the form

$$V(\mathbf{m}_t, x_t - bx_{t-1}) = F[M(\mathbf{m}_t^a, \mathbf{m}_t^c), x_t - bx_{t-1}]$$
$$= A[M(\mathbf{m}_t)](x_t - bx_{t-1}) - \frac{1}{2}B[M(\mathbf{m}_t)](x_t - bx_{t-1})^2$$

Future work

- We will measure the risk adjustment empirically.
  - For any credit card service, the risk adjustment to its certainty equivalent user cost can be measured by its beta, which depends on the covariance between the interest rate on credit card transaction and on the wealth portfolio of the consumer.
- Introduce heterogeneous agents.
  - We will disaggregate the consumers who repay soon enough and do not owe any interest which is about twenty percent of the credit card consumers, and consumers who pay interest which is about eighty percent of the credit card consumers.

Conclusions

- We are expecting to find a larger risk adjustment for credit card services under intertemporally nonseparable utility functions.
- The interest rates on most monetary assets are low and not very volatile.