Measuring contagion risk in international banking

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Executive summary

Our aim is to improve the measurement of credit risk of a country, taking international capital flows into account.

To this aim we have developed a **multilayer network model**, based on a **probabilistic tensor decomposition**, and on **multivariate contagion** between CDS spreads.

The model improves CDS spread predictions, providing a measure of contagion which can be used to monitor expected losses.

Data

Banks' foreign claims on each sector: public, bank, non-bank; aggregated by country (BIS Consolidated Banking Statistics).

CDS spread data for each country: public sector (5-year sovereign CDS spread); banks (5-year CDS spreads average); non-banks (spread between yield on country corporate index and US Treasury).

Multilayer Network model

The model represents the amount of each national banking system's foreign claims along three dimensions: bank, official and non bank private sectors.

We let a generic element (x_{ijk}) of the tensor be the share of the total foreign claims (E) borrowed by sector k of country i (authority), from the national banking system j (hub).

Probabilistic tensor decomposition

Tensor elements can be transformed into conditional frequencies:

$$h_{i|jk} = \frac{x_{ijk}}{\sum_{j=1}^{J}\sum_{k=1}^{K} x_{ijk}} \qquad i = 1, \ldots, I$$

$$a_{j|ik} = \frac{x_{ijk}}{\sum_{i=1}^{I}\sum_{k=1}^{K} x_{ijk}} \qquad j = 1, \ldots, J$$

$$r_{k|ij} = \frac{x_{ijk}}{\sum_{i=1}^{I}\sum_{j=1}^{J} x_{ijk}} \qquad k = 1, \ldots, K$$

which can estimate the transition probabilities of a Markov Chain:

$$\Pr[X_\tau = i | Y_\tau = j, Z_\tau = k]$$

$$\Pr[Y_\tau = j | X_\tau = i, Z_\tau = k]$$

$$\Pr[Z_\tau = k | Y_\tau = j, X_\tau = i].$$

We then let:

$$\Pr[X_\tau = i] = \sum_{j=1}^{J}\sum_{k=1}^{K} h_{i|jk} \Pr[Y_\tau = j, Z_\tau = k]$$

$$\Pr[Y_\tau = j] = \sum_{i=1}^{I}\sum_{j=1}^{K} a_{j|ik} \Pr[X_\tau = i, Z_\tau = k]$$

$$\Pr[Z_\tau = k] = \sum_{i=1}^{I}\sum_{j=1}^{J} r_{k|ij} \Pr[Y_\tau = j, X_\tau = i].$$

whose limits will be used as hub, authority and type probability scores:

$$u_i = \lim_{\tau\to\infty} \Pr[X_\tau = i]$$

$$v_j = \lim_{\tau\to\infty} \Pr[Y_\tau = j]$$

$$w_k = \lim_{\tau\to\infty} \Pr[Z_\tau = k],$$

Thus approximating the tensor with the outer product **M= u * v**

Multivariate contagion

Let $\mathbf{C}^k$ be the vector containing the CDS spreads of all countries, for a sector k, and **C** its average over sectors.
We define a **multivariate spread**, on the borrowing (SB) and on the lending side (SL), as:

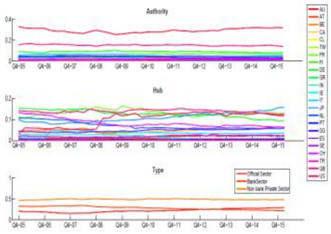
$$\mathrm{SB} = \mathbf{C} + \mathbf{C}^b \sum_{\mu=1}^{\Delta} \beta^{b,\mu} \mathbf{M}^\mu + \mathbf{C}^b \beta^o \mathbf{M} + \mathbf{C}^b \beta^p \mathbf{M}$$

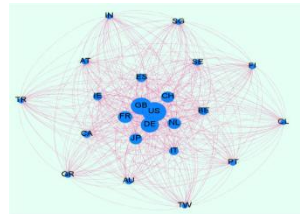
$$\mathrm{SL} = \mathbf{C} + \mathbf{C}^b \sum_{\mu=1}^{\Delta} \beta^{b,\mu} \left(\mathbf{M}^T\right)^\mu + \mathbf{C}^o \beta^o \mathbf{M}^T + \mathbf{C}^p \beta^p \mathbf{M}^T,$$

where b, o, p indicate the bank, official and non bank sectors , and:

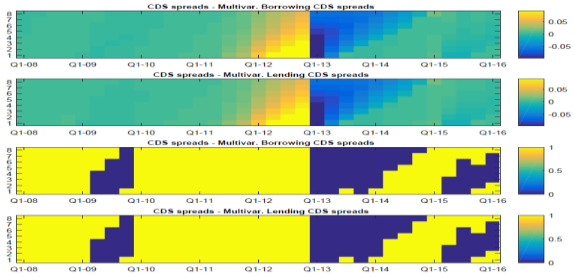
$$\alpha^o = w_o, \alpha^b = w_b, \alpha^p = w_p \qquad \beta^k = \frac{\alpha^k}{\lambda_1(\mathbf{M})},$$

Model estimates



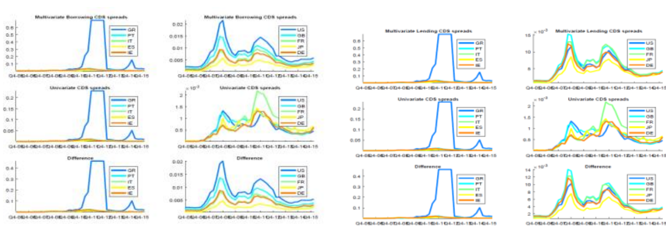


Predictive performance



Contagion effects

	Mean Univ. Cds	Mean Multiv. Borr. Cds	Mean Multiv. Lend. Cds
GR	0.0373	0.1120	0.1119
PT	0.0030	0.0092	0.0091
IT	0.0016	0.0064	0.0062
ES	0.0017	0.0064	0.0066
IE	0.0025	0.0081	0.0080
US	0.0006	0.0080	0.0051
GB	0.0006	0.0060	0.0060
FR	0.0007	0.0044	0.0058
JP	0.0006	0.0026	0.0038
DE	0.0005	0.0042	0.0050



Expected losses

Borrowing Side

$$EL_i^B = E * \sum_j \mathbf{M}_{ij} * SB_j.$$

Lending Side

$$EL_i^L = E * \sum_j \mathbf{M}_{ij}^T * SL_j$$