The Treatment of Financial Transactions in the SNA: A User Cost Approach

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Abstract

The paper considers some of the problems associated with the indirectly measured components of financial service outputs in the System of National Accounts (SNA), termed FISIM (Financial Intermediation Services Indirectly Measured). The paper considers how to integrate financial transactions into the balance sheet and production accounts of a firm; i.e., the paper looks at FISIM more broadly. In order to minimize the role of imputations, the paper considers a firm that raises capital at the beginning of the accounting period, engages in some form of productive activity during the period and then distributes the initial capital and any profits back to the capitalists who financed the firm.

1. Introduction

When financial transactions are introduced into a national income accounting framework, several problems are encountered:

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- Financial transactions are by definition in nominal currency units and hence there are difficulties in determining appropriate deflators to transform these monetary transactions into real components.
- It is difficult to determine what the appropriate discount rate is for each firm in the economy. We think of the firm's discount rate as a factor that converts transactions at the beginning of the accounting period into comparable units at the end of the accounting period.
- When user costs (and supplier benefits) are introduced into the accounting framework, the resulting user costs do not match up with the corresponding supplier benefit terms on the other side of the market, leading to a lack of additivity in the accounts (unless each firm uses the same discount rate).[2]
- If we try to avoid user cost imputations and just live with actual firm transactions, then we do not obtain the "right" user costs for "physical" capital services or the "right" user costs for demand deposits.

It is evident that the existing national income accounting framework does not provide a satisfactory framework for integrating firm financial transactions into the usual production accounts. In this paper, we will attempt to address some of these difficult accounting problems.[3]

Our approach will be to develop an accounting framework that starts out with actual firm transactions and take that approach as far as possible without introducing any extraneous imputations. In order to minimize the role of imputations, we think of an accounting period that corresponds to a fifteenth century merchant trading voyage, where at the beginning of the accounting period, the firm raises financial capital and uses the financial capital to purchase a ship and inventories of goods (this corresponds to the firm's beginning of the period "physical" capital stock). The voyage takes place and various revenues are generated by the sale of the goods at the destination port and various costs are incurred in purchasing intermediate inputs of goods at the destination port as well as the labour inputs associated with the voyage. Further revenues are generated by the sales of the goods purchased abroad at the home port. These sales and purchases of goods and labour payments generate the firm's *cash flow* or more accurately, the firm's *gross operating surplus*.[4] Finally, at the end of the return voyage, the ship is sold and the net proceeds of the voyage are distributed back to the investors in the voyage. Of course, for real life firms that undertake operations for multiple accounting periods, the accounting is more complex due to the difficulties associated with valuing the firm's capital stocks at the end of each accounting period and so imputations for these valuations must be made. Our focus on voyage or venture accounting eliminates this extra layer of imputations.

A brief outline of the paper is as follows.

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[2] See Diewert, Fixler and Zieschang (2013a) (2013b) on this point.

[3] Earlier work on introducing financial transactions into the system of accounts and more generally into the theory of the firm include Hancock (1985) (1991), Barnett (1987), Barnett and Zhou (1994) and Keuning (1999). For criticisms of the *System of National Accounts 2008* (see Eurostat, IMF, OECD, UN and the World Bank (2008)) treatment of financial sector outputs and inputs, see Hill (1996) and Sakuma (2013).

[4] Gross operating surplus less net interest payments equals cash flow.

Section 2 develops a stylized accounting framework for a non financial firm. The model of firm behavior basically follows that of Edwards and Bell (1961) and Hicks (1961) where the accounting period is decomposed into three parts: (i) the beginning of the period; (ii) the time period between the beginning and the end of the accounting period and (iii) the end of the accounting period. At the beginning of the period, the firm raises financial capital and purchases durable inputs. In the middle of the period, the firm produces outputs and uses intermediate and labour inputs. At the end of the period, the firm sells its (depreciated) durable inputs and returns the borrowed financial capital with interest payments and returns to equity financing. Our attention in this section is focussed on the firm's *gross operating surplus* which is equal to the value of outputs produced less intermediate and labour inputs used during the accounting period. We provide some preliminary decompositions of gross operating surplus into various payments to factors of production in this section.

In section 3, we introduce the concept of a *reference rate of interest*, which we later specify as the average weighted cost of capital for the firm. Using the reference rate of interest and various accounting identities, we are able to decompose the firm's gross operating surplus into more meaningful analytical terms. Two of these terms are the firm's *user cost of nonfinancial* (or physical) *capital* and the *user cost of holding demand deposits* (or money).

In section 4, we generalize our initial accounting framework in order to deal with the firm's holding of very liquid assets (near money) and the granting of trade credit. We develop a model which turns out to be a version of *Barnett's* (1980) *Divisia monetary assets model*. The problems associated with the deflation of financial aggregates into real components is also addressed in this section.

In section 5, we consider alternative approaches to the choice of the reference rate. The two choices we consider in this section are the *safe interest rate* and the *balancing rate of return* that is often used in productivity studies.

Sections 6 and 7 consider the recently developed *multiple reference rate methodologies* that are due to Wang and her coauthors (section 6) and to Zieschang (section 7).

Section 8 concludes with a brief listing of some of the unresolved issues associated with measuring the contribution of financial flows in production theory.

2. The Accounting Basics

We first consider the transactions that take place at the beginning of the accounting period. We assume that there are two classes of investor: one class that demands more security for their financial investments in the firm (these are the bond investors) and a second class that is willing to take more risk (these are the equity investors). The bond investors invest the amount V_B^0 at the beginning of the accounting period and expect to

earn the rate of return r_B^0 at the end of the accounting period. The equity investors invest the amount V_E^0 and expect to earn the rate of return r_E^0 where $r_E^0 > r_B^0$.[5]

Thus there is an inflow of dollars into the bank account of the firm at the beginning of the period equal to $V_B^0 + V_E^0$. How are these dollars allocated? We assume that some of the inflow dollars are held in the firm's deposit account and denote this amount by V_D^0.[6] Deposit accounts pay a low rate of interest equal to $r_D^0 < r_B^0 < r_E^0$. Some of the beginning of the period inflow dollars are invested in other securities or direct ventures. Denote the value of these investment dollars by V_I^0 and these investments are expected to earn the rate of return r_I^0. Finally, the remaining inflow dollars are allocated to the purchase of (physical) capital: we suppose that K^0 units of capital are purchased at the price P_K^0. We will denote the *inflow of dollars less the outflow at the beginning of the accounting period* by π^0. Under our assumptions, this net inflow of dollars is equal to 0; i.e., we have:[7]

(1) $\pi^0 \equiv V_B^0 + V_E^0 - P_K^0 K^0 - V_D^0 - V_I^0 = 0.$

Note that π^0 is also equal to the beginning of the period value of liabilities, $V_B^0 + V_E^0$, less the beginning of the period value of assets, $P_K^0 K^0 + V_D^0 + V_I^0$.

At the end of the accounting period, the firm will have accumulated the *Gross Operating Surplus* GOS^1. This is equal to the value of revenues generated by the firm during the accounting period, less the value of intermediate inputs less the value of labour service payments.[8] Since there are no major accounting difficulties with the components of Gross Operating Surplus, we will not provide a detailed breakdown of these components.

We will now consider the inflows and outflows of dollars at the end of the accounting period. GOS^1 is the first component of the inflows. The second component, $P_K^1(1-\delta)K^0$, is the sale of the depreciated capital stock, where P_K^1 is the end of period price of a new unit of the capital stock and δ is the depreciation rate. The third component, $V_D^0(1+r_D^0)$, is the value of the firm's initial stock of deposits, V_D^0, plus the interest paid by the bank on these deposits, $r_D^0 V_D^0$. The fourth component, $V_I^0(1+r_I^0)$, is the value of the firm's investments in other financial assets, V_I^0, (this term is the repatriation of the capital invested at the beginning of the period) plus the return earned on these investments, $r_I^0 V_I^0$. The fifth component is the repayment of the capital borrowed from bond holders plus the

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[5] The difference in these expected rates of return is regarded as a risk premium. Later, we will note that it is possible to regard r_B^0 and r_E^0 as ex post rates of return rather than expected rates of return.

[6] For simplicity, we assume that these deposits are held to the end of the accounting period. The analysis needs to be extended to include asset and inventory transactions that take place within the accounting period. The analysis in Diewert (2005a) which dealt with the integration of nonfinancial inventory transactions could be extended to the present framework.

[7] The student of accounting will recognize that we are essentially taking a double entry bookkeeping approach to the transactions of the firm, except that all of the transactions that take place between the beginning and the end of the accounting period are deferred until the end of the accounting period.

[8] Note that we are assuming that all of the flow transactions within the accounting period are realized at the end of each period. This is consistent with traditional accounting treatments of assets at the beginning and end of the accounting period and cash flows that occur during the period; see Peasnell (1981; 56).

interest earned by these bond investors, $-V_B^0(1+r_B^0)$. This item is a cash outlay and so it has a negative sign in front of it. The sixth component is the return of the capital borrowed from equity providers of funds plus the interest or dividend income earned by these equity investors, $-V_E^0(1+r_E^0)$. This item is also a cash outlay and so it has a negative sign in front of it. Finally, after all the above outflows are subtracted from the above inflows, the firm may earn a pure profit at the end of the period. This *end of period pure profit* π^1 is defined as the above cash inflows less the above cash outflows:[9]

$$(2)\ \pi^1 \equiv GOS^1 + P_K^1(1-\delta)K^0 + V_D^0(1+r_D^0) + V_I^0(1+r_I^0) - V_B^0(1+r_B^0) - V_E^0(1+r_E^0).$$

We will now take an end of period or ex post perspective and assume that we are at the end of the accounting period and GOS^1, P_K^1, δ, and of the rates of return which appear in (2) are known.[10] If π^1 is positive, then the firm makes a profit on its operations for the accounting period and this pure profit will be distributed back to the equity owners as a premium to their expected rate of return r_E^0. If π^1 is negative, then the equity owners will not make their "required" ex ante rate of return and the ex post actual rate of return can be obtained by setting π^1 equal to 0 and solving for the resulting r_E.

In principle, all of the transactions that are listed on the right hand sides of (1) and (2) have counterparts in the rest of the economy and so if we kept track of all financing decisions, interest flows in addition to the usual input and output flows in the production accounts of a system of national accounts, we could construct an expanded set of production accounts that included financial transactions which would add up; i.e., every transaction for a single sector in the expanded accounts would show up as a transaction in another sector of the accounts. There would be no lack of additivity problem in such a set of expanded accounts. The problem with such a set of accounts is that the transactions on the right hand side of (2) look rather unfamiliar! Thus in what follows, we will attempt to transform (2) into a more familiar set of transactions. In particular, we would like the user cost of nonfinancial capital to show up on the right hand side of (2).

Since the beginning of the period value of liabilities equals the corresponding value of assets (recall equation (1) above), we can add the right hand side of (1) to the right hand side of (2) and we obtain the following alternative expression for π^1:

$$(3)\ \pi^1 = GOS^1 - \delta P_K^1 K^0 + (P_K^1 - P_K^0)K^0 + r_D^0 V_D^0 + r_I^0 V_I^0 - r_B^0 V_B^0 - r_E^0 V_E^0.$$

Now equation (3) can be reorganized to give us a decomposition of the firm's gross operating surplus, GOS^1, in terms of pure profits π^1 and the other terms on the right hand side of (3):

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[9] All of the assets that appear on the right hand side of (2) could be disaggregated into multiple asset types for each broad category but this generalization is left to the reader. In the context of our voyage accounting model, r_E^0 would be the return to equity capital that just made π^1 equal to zero. In a more general model, r_E^0 would equal the rate of return on equity capital that was anticipated at the beginning of the accounting period and π^1 would represent *unanticipated profits* or *pure profits* above and beyond the rate that is required to induce equity holders to provide financial capital to the firm.

[10] It will be difficult to determine the required rate of return on equity capital, r_E^0.

(4) $GOS^1 = \pi^1 + [\delta P_K^1 - (P_K^1 - P_K^0)]K^0 - r_D^0 V_D^0 - r_I^0 V_I^0 + r_B^0 V_B^0 + r_E^0 V_E^0$.

The terms in square brackets on the right hand side of (4) can be recognized as part of *the user cost of capital services* except that the imputed interest rate term is missing; i.e., δP_K^1 is the *depreciation term* and $- (P_K^1 - P_K^0)$ is the *revaluation term* in the usual user cost of capital. However, the remaining terms on the right hand side of (4) look unfamiliar. But it is true that the right hand side of (4) gives us *an explicit decomposition of the gross operating surplus of the firm into explanatory factors* where the financing decisions of the firm figure prominently in this decomposition.[11]

3. The Reference Rate and Analytic Decompositions of Gross Operating Surplus

Our goal in the remainder of the paper is to obtain *useful decompositions* of the firm's gross operating surplus into explanatory terms that make sense. In order to make further progress, we now make a somewhat arbitrary assumption. From equation (1), we know that the value of liabilities at the beginning of the accounting period equals the corresponding value of assets. Hence we can multiply the initial stock of liabilities less assets by the *reference interest rate* r_R^0 and obtain the following equation:

(5) $[V_B^0 + V_E^0 - P_K^0 K^0 - V_D^0 - V_I^0] r_R^0 = 0$.

The arbitrary element in equation (5) is the choice of the reference interest rate, r_R^0: at this stage of the analysis, this rate is completely arbitrary and yet, as we will see, it will play a key role in what follows.

Now add the left hand side of equation (5) to the right hand side of equation (3) and we obtain the following expression for π^1:

(6) $\pi^1 = GOS^1 - [r_R^0 P_K^0 + \delta P_K^1 - (P_K^1 - P_K^0)]K^0 - (r_R^0 - r_D^0)V_D^0 + (r_I^0 - r_R^0)V_I^0$
$- (r_B^0 - r_R^0)V_B^0 - (r_E^0 - r_R^0)V_E^0$.

Equation (6) can be rearranged to give an *alternative exact decomposition of the firm's end of period gross operating surplus* into explanatory terms:

(7) $GOS^1 = \pi^1 + [r_R^0 P_K^0 + \delta P_K^1 - (P_K^1 - P_K^0)]K^0 + (r_R^0 - r_D^0)V_D^0 - (r_I^0 - r_R^0)V_I^0$
$+ (r_B^0 - r_R^0)V_B^0 + (r_E^0 - r_R^0)V_E^0$.

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[11] If we simplify the accounts by absorbing the pure profits term into the ex post return on equity (i.e., set $\pi^1 = 0$ and use equation (2) or (3) to solve for the balancing rate of return on equity that makes the equation equal to zero), then all of the terms on the right hand side of (4) will have offsetting entries elsewhere in an expanded set of accounts. When we subtract depreciation from gross operating surplus, we obtain net operating surplus. The placement of the revaluation term is more controversial; if the price of the asset declines over time due to technical progress, then the revaluation term could be regarded as an obsolescence charge and could be added to wear and tear depreciation. However, if the price of the asset increases over time, then the revaluation term typically shows up in the revaluation accounts of the System of National Accounts. But the basic point here is that there is no additivity problem in principle with the expanded system of accounts when we use the decomposition of gross operating surplus given by (4).

The expression in square brackets on the right hand side of (7) can be recognized as the user cost of capital services.[12] Note that the interest rate component of this user cost, $r_R^0 P_K^0$, uses the reference rate r_R^0 to value the *opportunity cost* of tying up the firm's financial capital in holding physical capital. With this observation, the choice of the reference rate is no longer so arbitrary: *the reference rate can be interpreted as the interest rate that represents waiting services in the firm's user cost of capital.*[13]

Thus equation (7) provides a decomposition of the firm's gross operating surplus into the sum of the following components:

- the *pure profits* earned by the firm during the accounting period, π^1;
- the value of *nonfinancial capital services*, $[r_R^0 P_K^0 + \delta P_K^1 - (P_K^1 - P_K^0)]K^0$;
- the *user cost of holding demand deposits* during the period, $(r_R^0 - r_D^0)V_D^0$;
- (less) the *net margins* earned by the firm on its financial investments, $- (r_I^0 - r_R^0)V_I^0$ (this is the firm counterpart to *loan margins* earned by banks on their loan portfolios) and
- the sum of two terms, $(r_B^0 - r_R^0)V_B^0 + (r_E^0 - r_R^0)V_E^0$, which reflect the costs of raising financial capital via debt and equity capital, $r_B^0 V_B^0 + r_E^0 V_E^0$, relative to raising the same amount of financial capital at the reference rate, $r_R^0 V_B^0 + r_R^0 V_E^0$.

Typically, the reference rate r_R^0 will lie between the debt interest rate r_B^0 and the required equity rate of return r_E^0. Under these conditions, $(r_B^0 - r_R^0)V_B^0$ will be negative and $(r_E^0 - r_R^0)V_E^0$ will be positive. Thus the positive term $(r_E^0 - r_R^0)$ can be interpreted as a *positive equity premium* that is earned by equity capital for taking on more risk and the negative term $(r_B^0 - r_R^0)$ can be interpreted as a *negative debt discount* to reflect the lower risk that is associated with the provision of debt capital. Alternatively, $r_E^0 - r_R^0$ can be interpreted as the *user cost* of raising financial capital via equity financing, relative to the average cost of raising funds and since $r_B^0 - r_R^0 = - (r_R^0 - r_B^0)$, $r_R^0 - r_B^0$ can be interpreted as the *supplier benefit* [14] to the firm of raising financial capital via debt financing.

A natural choice for the reference rate is r_C^*, the *average cost of raising financial capital from debt and equity financing*;[15] i.e., define r_C^* (the average cost of funds interest rate) as follows:

$$(8) \quad r_C^* \equiv [r_B^0 V_B^0 + r_E^0 V_E^0]/[V_B^0 + V_E^0].$$

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[12] For a sampling of material on the user cost of capital concept, the reader is referred to Hall and Jorgenson (1967), Jorgenson and Griliches (1967), Christensen and Jorgenson (1969), Diewert (1974) (1980) (2005a) (2005b), Harper, Berndt and Wood (1989), Jorgenson (1989) (1996a) (1996b), Hulten (1990) (1996), Diewert and Lawrence (2000) and Schreyer (2001) (2009).

[13] Thus $r_R^0 P_K$ represents the waiting services term in the user cost of capital; see Rymes (1968) (1983) on the concept of waiting services.

[14] See Diewert, Fixler and Zieschang (2013a) for the introduction of the term "supplier benefit" as a term for a negative user cost.

[15] Inklaar (2010) used this reference rate in his study of U.S. productivity. His study used a methodology that is similar to ours except he focused on adding various intangible assets to his asset base rather than adding monetary assets to the nonreproducible asset base.

Replacing the general reference rate r_R^0 in (7) by r_C^* leads to the following decomposition of gross operating surplus:

$$(9) \quad GOS^1 = \pi^1 + [r_C^* P_K^0 + \delta P_K^1 - (P_K^1 - P_K^0)]K^0 + (r_C^* - r_D^0)V_D^0 - (r_I^0 - r_C^*)V_I^0.$$

Thus the last two terms on the right hand side of (7) have vanished on the right hand side of (9)[16] and so when we set the reference rate equal to the firm's average cost of financial funds, we find that gross operating surplus is equal to pure profits π^1, plus the value of nonfinancial capital services $[r_C^* P_K^0 + \delta P_K^1 - (P_K^1 - P_K^0)]K^0$ plus the cost of deposit services $(r_C^* - r_D^0)V_D^0$ less margins on financial investments and loans $- (r_I^0 - r_C^*)V_I^0$. This seems to be a satisfactory analytical decomposition of gross operating surplus for a non banking firm.[17] However, other choices for the reference rate are possible as we shall see in the next two sections.

The question of where to place the last two terms on the right hand side of (9), $(r_C^* - r_D^0)V_D^0 - (r_I^0 - r_C^*)V_I^0$, in a national income accounting framework now arises; i.e., should these terms be moved out of the income side of the accounts into the production accounts (the output and intermediate input part of the accounts)? The first term is the imputed value of deposit services and the second term is the negative of loan and investment margins. Since the provision of deposit services by banks is generally regarded as an output in the SNA, consistency would suggest that the first term be moved out of the income accounts and into the intermediate input part of the accounts. Similarly, since bank loan services are generally regarded as a banking sector output, consistency across sectors would suggest that the last term be moved into the output part of the accounts.[18] This is a sensible strategy but it would be useful to distinguish these new rows of the production accounts as financial outputs and inputs that require special treatment. The special nature of these financial transactions is due to the following factors:

- There are no natural deflators for the entries in these financial rows and so users need to be alerted to the fact that the corresponding real or volume entries will necessarily be somewhat arbitrary.
- We cannot expect these entries for a specific firm or sector to be offset by another entry in the accounts that is equal in magnitude but opposite in sign to the entries in these financial outputs and inputs; i.e., additivity will in general be lost for the rows in the production accounts that correspond to these financial outputs and inputs.[19]

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[16] Of course, these two missing terms (which sum to zero when the reference rate is defined by (8)) can be brought back onto the right hand side of (9) if this is desired for some analytic purpose but the decomposition given by (9) seems to be very suitable for production function studies of the firm.

[17] There is a similar decomposition for a banking firm but the cost of deposit services term changes sign into a benefit of creating deposits; see Diewert, Fixler and Zieschang (2012b) for the details.

[18] It should be noted that this move to achieve consistency with the treatment of banking sector loans would imply a large change to the present structure of the SNA and is unlikely to be implemented without more study of the implications of the change.

[19] See Diewert, Fixler and Zieschang (2013a) (2013b) for an elaboration of this point. If the reference rate is chosen to be the same across all sectors in the system of accounts, then additivity can be restored.

In the following section, we will extend the above model by decomposing the value of firm financial investments, V_I, into two components: one component which has a low rate of return associated with it and another which has a higher rate of return.

4. Barnett's Monetary Aggregates and the Deflation Problem

Barnett (1980) worked out a nice theory of monetary aggregation that applied to households. He noted that very liquid assets could serve as a fairly close substitute for deposits and hence broader measures of monetary holdings could be derived by applying modern index number theory and forming broader monetary aggregates. To apply his framework in our present firm context, we need to decompose the firm's holdings of financial investments, V_I^0, into at least two components:[20]

- Holdings V_{IL}^0 of a very liquid asset that earns the *low* interest rate r_{IL}^0 which is less than the reference rate r_R^0 and
- Holdings V_{IH}^0 of a risky asset that earns the *high* interest rate r_{IH}^0 which is greater than the reference rate r_R^0.

The very liquid assets V_{IH}^0 can be regarded as part of the firm's *working capital*, along with its holdings of demand deposits, V_D^0.

Using the decomposition of V_I^0 into V_{IL}^0 plus V_{IH}^0, equations (3), (5) and (7) become the following equations:

$$(10)\ \pi^1 = GOS^1 - \delta P_K^1 K^0 + (P_K^1 - P_K^0)K^0 + r_D^0 V_D^0 + r_{IL}^0 V_{IL}^0 + r_{IH}^0 V_{IH}^0 - r_B^0 V_B^0 - r_E^0 V_E^0\ ;$$

$$(11)\ [V_B^0 + V_E^0 - P_K^0 K^0 - V_D^0 - V_{IL}^0 - V_{IH}^0]r_R^0 = 0\ ;$$

$$(12)\ GOS^1 = \pi^1 + [r_R^0 P_K^0 + \delta P_K^1 - (P_K^1 - P_K^0)]K^0 + (r_R^0 - r_D^0)V_D^0 + (r_R^0 - r_{IL}^0)V_{IL}^0$$
$$- (r_{IH}^0 - r_R^0)V_{IH}^0 + (r_B^0 - r_R^0)V_B^0 + (r_E^0 - r_R^0)V_E^0.$$

Equation (12) is the new decomposition of gross operating surplus into analytical components. Under our assumptions on interest rates, the terms $(r_R^0 - r_D^0)V_D^0$ and $(r_R^0 - r_{IL}^0)V_{IL}^0$ will both be positive and it is evident that these terms represent the opportunity costs (relative to the cost of capital r_R^0) of holding the amount V_D^0 in demand deposits and the amount V_{IL}^0 in low yielding, liquid investments throughout the period. The two terms are in nominal dollar units and in order to apply index number theory to these two components of broadly defined monetary services, we need to decompose these two value flows into price and quantity components. Let ρ_D^0 and ρ_{IL}^0 be appropriate deflators for these two value flows. Then the prices and quantities of the two components of monetary services are defined as follows:

$$(13)\ P_D^0 \equiv (r_R^0 - r_D^0)\rho_D^0\ ;\ P_{IL}^0 \equiv (r_R^0 - r_{IL}^0)\rho_{IL}^0\ ;\ Q_D^0 \equiv V_D^0/\rho_D^0\ ;\ Q_{IL}^0 \equiv V_{IL}^0/\rho_{IL}^0\ .$$

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[20] Of course, V_I^0 can be further decomposed into many assets, including accounts receivable (or trade credit).

Barnett (1980; 17) used the same true cost of living index (or alternatively, a consumer price index could be used) to deflate all of his household nominal monetary variables into real variables. In our firm context, it is not so clear what the appropriate deflators, ρ_D^0 and ρ_{IL}^0, should be. We will discuss this choice problem below. Given the prices and quantities of monetary assets defined by (13), we can follow Barnett (1980; 39) and use a superlative index number formula to construct a monetary aggregate for the two assets.[21]

How should the asset deflators ρ_D^0 and ρ_{IL}^0 be chosen? There is no unambiguous answer to this question. If average stocks of monetary balances are being held in order to make payments to variable inputs and to fund purchases of inventory stocks and other capital input purchases, then a price index ρ_X^0 for the value of input purchases during the period would be an appropriate deflator for the firm's holdings of deposits and other near monetary stocks. What deflator should be used to deflate the firm's high yielding investments, V_{IH}^0? One could argue that the real cost of making these investments is the fact that money spent on risky investments cannot be spent on input purchases and hence the same input price index ρ_X^0 could be used as a deflator for these risky investments. Another alternative would be to deflate all financial nominal amounts by a suitable consumer price index ρ_C^0. The justification for this alternative would be to measure the real value of a monetary unit in terms of a representative consumption bundle or more generally, in terms of a cost of living index for a reference population.

Obviously, the above paragraph on deflation of monetary flows is very incomplete. Basu (2009) summed up the unsatisfactory treatment of financial variables in economic theory as follows:

"No method of measuring financial sector prices (and hence real output) has yet commanded a consensus. In fact, there is even disagreement about how to measure *nominal* output in one of the most important financial sectors, namely banking. Thus, it is not surprising that I shall propose different answers than Fixler to the questions that he raises. But more important than the specifics of any particular issue is a general contention: in economics, when a conceptual disagreement has lasted a long time with no resolution in sight, it is usually a sign that economic theory has not been applied sufficiently rigorously. The only way to make progress in this area is to start from detailed models of what financial institutions actually do, and the market environment in which they operate. Once that is done, the measurement implications are usually obvious in principle, although the implied measures may be exceedingly difficult to implement in practice." Susanto Basu (2009; 267).

We conclude this section with a further cautionary note: we have not modeled the riskiness of alternative financial investments in a completely rigorous way.[22] However, until the theory of firm behaviour under uncertainty with explicit modeling of the firm's financing decisions has been developed to the extent that there is an accepted consensus

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[21] See Diewert (1976) for the definition of a superlative index. Barnett (1980; 39) for his household example used the Fisher and Törnqvist superlative indexes and found that the two formula gave identical results to three decimals and commented that "the choice between these two indices is of no importance".

[22] See Wang (2003), Barnett and Wu (2005) and Wang, Basu and Fernald (2009) for rigorous approaches to the treatment of uncertainty in a user cost context. However, a consensus on the "right' approach to the treatment of uncertainty in a national income accounting framework has not yet emerged.

on how to proceed, we will have to make do with incomplete modeling. Since there is an urgent need to develop an adequate accounting framework for measuring financial outputs and inputs in a national income accounting framework, we hope that the approaches explored in this paper will be useful in forming a consensus on how to proceed at a practical level.

In the following sections of this paper, we revert back to the more aggregated model of firm behavior that was described in sections 2 and 3 above. The subsequent discussion will focus on alternative choices for the reference rate and on generalizations of the model in section 3 to include multiple reference rates.

5. Alternative Choices for the Reference Rate

There are advantages in assuming that there is only a single reference rate r_R^0 for the firm, since this assumption leads to a useful interpretation for the firm's end of the period profits. Using definitions (1) and (2) (which define the cash transactions of the firm at the beginning and end of the period) and the assumption (5) of a single reference rate, it be seen that the firm's end of period profits can be written as follows:

(14) End of period profits $= \pi^0(1+r_R^0) + \pi^1$.

Thus if the firm chooses inputs and outputs to maximize the right hand side of (14), this will be equivalent to the maximization of discounted cash flows; i.e., (14) is equivalent to the maximization of $\pi^0 + (1+r_R^0)^{-1}\pi^1$. The maximization of discounted cash flows is the traditional approach to intertemporal production theory.[23]

In section 3, we considered the implications of choosing the reference rate equal to the average cost of raising debt and equity financial capital. Another possible choice is the *safe interest rate*, r_S^*. This rate would correspond to the yield on triple A rated assets or on bond rates for short term government securities (for a country with a suitably high debt rating). Inserting this choice of reference rate into (7) leads to the following decomposition of the firm's gross operating surplus:

(15) $GOS^1 = \pi^1 + [r_S^* P_K^0 + \delta P_K^1 - (P_K^1 - P_K^0)]K^0 + (r_S^* - r_D^0)V_D^0 - (r_I^0 - r_S^*)V_I^0$
$\qquad + (r_B^0 - r_S^*)V_B^0 + (r_E^0 - r_S^*)V_E^0$.

Comparing the decomposition given by (15) with our earlier decomposition (9) which used the cost of funds reference rate r_C^*, it can be seen that we now have an extra two terms, namely $(r_B^0 - r_S^*)V_B^0$ and $(r_E^0 - r_S^*)V_E^0$. Both of these terms will generally be positive since the safe rate of return will generally be below the bond and equity interest rates. The question is: what should we do with these two terms? Should they be left in the

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[23] See Hicks (1939). See Edwards and Bell (1961), Hicks (1961), Diewert (1980) (2010; 760-762) and Diewert, Fixler and Zieschang (2013b) for the specialization of this theory to the case of a single period. Diewert (2010; 760) called this the Neo-Austrian approach to producer theory and noted that by recognizing goods in process, a general intertemporal production model can be reduced to a sequence of one period problems.

income part of the accounts or should they be shifted into the production accounts where they would appear as sectoral intermediate input costs. The latter treatment seems to be a logical one if we have shifted loan and investment margins into the production accounts since the last two terms in (15) are similar in nature (but of course, they will generally have the opposite sign to loan margins). The major advantage of choosing the reference rate to be the safe interest rate is that the various margins and user costs on the right hand side of (15) will have offsetting entries in other parts of the system of national accounts so that additivity of the system can be preserved. However, a possible disadvantage of the choice of the safe rate as the reference rate is that as compared with the choice of the cost of funds rate r_C^* as the reference rate, the value of capital services and of deposit services will be dramatically reduced and the value of loan and investment margin services, $(r_I^0 - r_S^*)V_I^0$, will be dramatically increased. Finally, the user costs of raising funds via debt and equity relative to raising funds at the safe interest rate, $(r_B^0 - r_S^*)V_B^0$ and $(r_E^0 - r_S^*)V_E^0$ respectively, will both become large and positive. These last two margins become large and positive at the cost of the capital services term becoming smaller and this is the difficulty with the use of the safe interest rate as the reference rate. Essentially, these last two terms can be interpreted as extra profits that the firm has to earn in order to cover its costs of raising financial capital. Thus we have shifted costs out of the user cost of capital and into these margin terms which seems to be a dubious strategy.

Another alternative strategy that is frequently used in order to determine the reference rate for a nonfinancial firm is to use the *balancing rate of return reference rate* r_{BR}^*, which is defined by assuming that $\pi^1 = 0$ and by solving the following equation which sets the user cost of capital times K^0 equal to the gross operating surplus:

(16) $GOS^1 = [r_{BR}^* P_K^0 + \delta P_K^1 - (P_K^1 - P_K^0)]K^0$ or
(17) $r_{BR}^* \equiv \{GOS^1 - [\delta P_K^1 - (P_K^1 - P_K^0)]K^0\}/P_K^0 K^0$.

Thus all of the financial transactions of the firm are suppressed in the decomposition of gross operating surplus that is given by (16). Now we want to compare the balancing rate of return r_{BR}^* with the cost of funds rate of return r_C^* defined by (8). When we set $\pi^1 = 0$, the cost of funds decomposition of gross operating surplus defined by (9) can be rewritten as follows:

(18) $GOS^1 - [\delta P_K^1 - (P_K^1 - P_K^0)]K^0] = (r_C^* - r_D^0)V_D^0 + (r_C^* - r_I^0)V_I^0$.

Using (17) and (18), it can be seen that we have the following relationship between r_{BR}^* and r_C^*:

(19) $r_{BR}^* = [(r_C^* - r_D^0)V_D^0 + (r_C^* - r_I^0)V_I^0]/P_K^0 K^0$.

Usually, a nonfinancial firm will hold demand deposits and since the deposit rate r_D^0 will almost always be well below the firm's average cost of capital r_C^*, it can be seen that the first term on the right hand side of (18) will generally be positive. A nonfinancial firm will typically not have substantial financial investments and if it does, usually the rate of return earned on these financial investments r_I^0 will be close to the firm's cost of capital

r_C^*. Thus typically, the right hand side of (18) will be positive and so the balancing rate of return will generally exceed the firm's cost of raising financial capital; i.e., typically

(20) $r_{BR}^* > r_C^*$.

Thus relative to the more accurate decomposition of gross operating surplus that is given by (9), the less accurate decomposition given by the usual balancing rate of return methodology (17) will have the following characteristics:

- The value of nonfinancial capital services will generally be overstated;
- The value of deposit services will be dramatically understated (since it will be set equal to zero) and
- The role of investment or loan margins will be missing.

The fact that deposit services are missing in traditional production function studies of the economy that use the balancing rate of return methodology is potentially large source of bias in these studies, since presently, many firms in developed economies are holding very large deposit balances.

6. Multiple Reference Rate Methodologies: The Wang Group Approach

Rather than assuming a single reference rate, it is possible to preserve the structure of firm cash flows by replacing assumption (5) by the following assumption which has multiple reference rates:

(21) $V_B^0 r_B^* + V_E^0 r_E^* - P_K^0 K^0 r_K^* - V_D^0 r_D^* - V_I^0 r_I^* = 0$.

Thus there are now five reference rates: r_B^*, r_E^*, r_K^*, r_D^* and r_I^* so that there is one reference rate for each type of asset and liability. Four of these rates can be chosen arbitrarily but the fifth rate must be chosen to satisfy equation (21).[24]

Now add the left hand side of equation (21) to the right hand side of equation (3) and we obtain the resulting expression for π^1:

(22) $\pi^1 = GOS^1 - [r_K^* P_K^0 + \delta P_K^1 - (P_K^1 - P_K^0)]K^0 - (r_D^* - r_D^0)V_D^0 + (r_I^0 - r_I^*)V_I^0$
$- (r_B^0 - r_B^*)V_B^0 - (r_E^0 - r_E^*)V_E^0$.

Equation (22) can be rearranged to give an *alternative exact decomposition of the firm's end of period gross operating surplus*:

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[24] This multiple reference rate methodology was introduced by Wang (2003). Papers which develop this methodology are Wang, Basu and Fernald (2009), Basu, Inklaar and Wang (2011), Colangelo and Inklaar (2012) and Inklaar and Wang (2012a) (2012b) and Wang and Basu (2012) (the Wang Group). These papers use the multiple reference rate methodology with the reference rate for nonfinancial capital being determined residually using a variant of equation (21). We need equation (21) to hold because when we add terms to the firm's actual cash flows, these additional terms must sum to zero so that the firm's cash flows remain unaffected.

$$(23)\ GOS^1 = \pi^1 + [r_K^* P_K^0 + \delta P_K^1 - (P_K^1 - P_K^0)]K^0 + (r_D^* - r_D^0)V_D^0 - (r_I^0 - r_I^*)V_I^0$$
$$+ (r_B^0 - r_B^*)V_B^0 + (r_E^0 - r_E^*)V_E^0.$$

Of course, the practical problem with the multiple reference rate methodology is: how exactly are the various reference rates to be determined? What principles are to be used in justifying a particular selection of rates?

The Wang Group want to avoid putting risk premiums into the outputs of the banking sector[25] so they choose reference rates for deposits and loans to be very close to the corresponding actual rates by choosing reference debt rates to match the various financial assets on the bank's balance sheet, where the reference rates have similar maturity and risk characteristics. Thus a bank's service outputs for the deposits it creates and the bank loans it makes should reflect the costs of servicing the various accounts.[26] The Wang Group worked out their methodology for a bank and it is not completely clear exactly how their methodology would apply to a nonfinancial firm. Applying their methodology to the right hand side of (23) might lead to the choice of a reference deposit rate r_D^* which is close to the actual deposit rate r_D^0 and to a reference investment (or loan) rate r_I^* which is slightly above the actual net loan rate (after loan losses) r_I^0. Typically they would choose the reference rates for bonds and equity, r_B^* and r_E^*, to be equal to the corresponding actual rates r_B^0 and r_E^0 and so the final reference rate for nonfinancial capital, r_K^*, would be determined by solving equation (21) for r_K.[27]

Suppose we accept the above assumptions so that we set $r_B^* = r_B^0$ and $r_E^* = r_E^0$ and we choose reference rates for deposits and other financial investments, r_D^* and r_I^*, that are close to the observed rates, r_D^0 and r_I^0 respectively. Define the *average reference rate of return on financial assets*, r_{FA}^*, as follows:

$$(24)\ r_{FA}^* \equiv [r_D^* V_D^0 + r_I^* V_I^0]/[V_D^0 + V_I^0].$$

Define the firm's beginning of the period *ratio of financial assets to nonfinancial assets* (physical capital), $\rho_{FA/K}$, as follows:

$$(25)\ \rho_{FA/K} \equiv [V_D^0 + V_I^0]/P_K^0 K^0.$$

Now substitute our assumptions on reference rates into equation (21) and solve for the *nonfinancial firm counterpart to the Wang Group reference rate for nonfinancial capital*, r_W^*:

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[25] This is reasonable: waiting services and risk assumption services can be regarded as primary inputs and hence the remuneration for the provision of these services belongs in the income accounts.

[26] Zieschang (2012) refers to these components of bank output as the "account servicing" components of bank output.

[27] Since the Wang Group has not explicitly addressed what reference rates they would choose for a non banking firm, we are engaging in a certain amount of guesswork on how they would choose their reference rates for a nonfinancial business.

(26) $r_W^* = [V_B^0 r_B^* + V_E^0 r_E^* - V_D^0 r_D^* - V_I^0 r_I^*]/P_K^0 K^0$
$= r_C^* + [r_C^* - r_{FA}^*]\rho_{FA/K}$

where r_C^* is the average cost of raising financial capital from debt and equity financing defined earlier by (8) and we have used (1) and definitions (24) and (25) in order to derive the second equation in (26). A "typical" nonfinancial firm will not have extensive investments, so usually, the average reference rate on financial assets r_{FA}^* will be close to the reference deposit rate r_D^* which in turn will be close to the reference deposit rate r_D^* which will be much lower than the average cost of capital r_C^* defined by (8). Thus the interest rate which will be imputed to physical capital using the Wang Group methodology, r_W^*, will typically be *larger* than the average cost of capital, r_C^*, since the ratio of financial assets to physical assets, $\rho_{FA/K}$, will always be positive.

Now substitute our Wang Group assumptions about reference rates into (23) and we obtain the following *exact decomposition* of the firm's end of period gross operating surplus:[28]

(27) $GOS^1 = \pi^1 + [r_W^* P_K^0 + \delta P_K^1 - (P_K^1 - P_K^0)]K^0 + (r_D^* - r_D^0)V_D^0 - (r_I^0 - r_I^*)V_I^0.$

The above decomposition of gross operating surplus is very similar to our earlier decomposition (9) which used a single reference rate, r_C^*, which was the average cost of raising financial capital via debt and equity financing. For easy reference, we repeat (9) as (28):

(28) $GOS^1 = \pi^1 + [r_C^* P_K^0 + \delta P_K^1 - (P_K^1 - P_K^0)]K^0 + (r_C^* - r_D^0)V_D^0 - (r_I^0 - r_C^*)V_I^0.$

Comparing the decompositions (27) and (28), under the assumption that r_W^* is less than r_C^*, it can be seen that the user cost of capital in (28) will be smaller than the corresponding user cost in (27). If the reference rates r_D^* and r_I^* in (27) are close to the observed rates, r_D^0 and r_I^0, then the last two terms on the right hand side of (27) will be close to zero whereas the last two terms on the right hand side of (28) will usually be much larger in magnitude. If we assume V_I^0 is equal to zero, then we can be more definite about the differences between the two decompositions: the cost of capital decomposition (28) will have a smaller contribution to gross operating surplus from the user cost of capital and a larger contribution to the firm's holdings of monetary assets.

A problem with the Wang Group methodology is that the assumptions about financial reference interest rates lead directly to an interest rate term that is applied to nonfinancial capital and this interest rate may be quite different from the usual interest rate that we insert into the user cost of capital, which is typically related to the cost of raising

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[28] The Wang Group decomposition of gross operating surplus given by (27) can be compared to our earlier decomposition of GOS using a balancing rate of return given by (16). If $\pi^1 = 0$, $r_D^* = r_D^0$ and $r_I^* = r_I^0$, then r_W^* will equal the balancing rate of return r_{BR}^* and the Wang Group decomposition (27) will collapse down to the balancing rate of return decomposition (16). Thus with profits equal to zero and the reference rates close to the actual rates, we would expect the Wang Group decomposition of GOS to be close to the balancing rate of return decomposition.

financial capital.[29] We now turn to an even more general multiple reference rate methodology that has the flexibility of the Wang Group with respect to pricing financial services but at the same time, can insert the "right" interest rate for the user cost of physical capital.

7. Multiple Reference Rate Methodologies: The Zieschang Approach

The methodology that will be described in this section is due to Zieschang (2013). Our derivation of his methodology is a bit different but it is completely equivalent, except we are considering nonfinancial firms whereas he considered only financial firms.[30]

Recall the single reference rate methodology that was described in section 3 above. Our starting point will be the decomposition of gross operating surplus that was given by equation (7). The basic insight of Zieschang was to decompose the various financial sector user costs and supplier benefit terms on the right hand side of (7) into two components:

- A component that represents the pure services aspect of the transactions associated with each user cost or supplier benefit which Zieschang interpreted as "account servicing components" of bank.
- Another component that represents some kind of financial intermediation services.

The account servicing components of Zieschang's user costs and supplier benefits are entirely similar to the Wang Group's notions of bank service outputs and inputs. Thus assume that we have determined *suitable reference rates* r_B^*, r_E^*, r_D^* and r_I^* that are close to the observed rates r_B^0, r_E^0, r_D^0 and r_I^0 and we also have determined a suitable overall reference rate r_R^* that we want to apply to the physical capital of the firm. Then applying the single reference rate r_R^* in the manner explained in section 3 above, the counterpart to (7), the decomposition of gross operating surplus at the end of the accounting period, is as follows:

$$
\begin{aligned}
(29)\ \mathrm{GOS}^1 &= \pi^1 + [r_R^* P_K^0 + \delta P_K^1 - (P_K^1 - P_K^0)]K^0 + (r_R^* - r_D^0)V_D^0 - (r_I^0 - r_R^*)V_I^0 \\
&\quad + (r_B^0 - r_R^*)V_B^0 + (r_E^0 - r_R^*)V_E^0 \\
&= \pi^1 + [r_R^* P_K^0 + \delta P_K^1 - (P_K^1 - P_K^0)]K^0 + (r_R^* - r_D^* + r_D^* - r_D^0)V_D^0 - (r_I^0 - r_I^* + r_I^* - r_R^*)V_I^0 \\
&\quad + (r_B^0 - r_B^* + r_B^* - r_R^*)V_B^0 + (r_E^0 - r_E^* + r_E^* - r_R^*)V_E^0 \\
&= \pi^1 + [r_R^* P_K^0 + \delta P_K^1 - (P_K^1 - P_K^0)]K^0 + (r_R^* - r_D^*)V_D^0 + (r_D^* - r_D^0)V_D^0 - (r_I^0 - r_I^*)V_I^0 \\
&\quad + (r_I^* - r_R^*)V_I^0 + (r_B^0 - r_B^*)V_B^0 + (r_B^* - r_R^*)V_B^0 + (r_E^0 - r_E^*)V_E^0 + (r_E^* - r_R^*)V_E^0
\end{aligned}
$$

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[29] A related problem is that the Wang Group imputation for deposit services will be much smaller than our preferred imputation $(r_C^* - r_D^0)V_D^0$ that we obtained in (9) for the (opportunity) cost of the firm's deposit services. Our preferred approach seems to be more consistent with Barnett's (1980) approach to the determination of the user cost for monetary services.

[30] This distinction is not important: nonfinancial firms are just like financial firms except that financial firms (banks) have the power to raise financial capital via the creation of demand deposits. Thus financial firms will have an extra liability term in the decomposition of their operating surplus.

where the second equation in (29) follows by adding and subtracting terms. The third equation in (29) gives the Zieschang decomposition of the firm's end of period gross operating surplus into explanatory terms. His decomposition does succeed in associating an appropriate reference interest rate r_R^* (which can be chosen to be the average cost of financial capital r_C^* defined earlier) but now we have a large number of account servicing terms on the right hand side of (29) plus the financial intermediation terms to interpret and allocate to the income accounts or the production accounts.[31]

Our conclusion at this point is that the Zieschang decomposition has a great deal of flexibility associated with it but at the same time, it is somewhat complex and not that easy to interpret. Choosing an appropriate constellation of reference rates also is problematic.

8. Conclusion

We have tried to integrate financial transactions into the traditional theory of the firm with the hope that such an integration would be helpful in developing a consistent system of national accounts. In particular, we showed that bringing in financial transactions into the traditional theory of the firm (which deals with inputs and outputs which have definite physical units of measurement as opposed to nominal financial values) can be viewed as the problem of decomposing gross operating surplus into analytically meaningful terms.

A large number of alternative decompositions of GOS were presented in the paper. At our present state of knowledge, the author feels that the decomposition given by equation (9) is the most suitable one. This decomposition involves the choice of a single reference rate, r_C^*, which is the firm's average cost of raising financial capital from debt and equity financing. The decomposition (9) is consistent with standard intertemporal production theory and requires fewer imputations than the multiple reference rate approaches due to the Wang Group and Zieschang.

Many problems associated with the integration of the firm's financial decisions with its "real" decisions remain unresolved. Some of these unresolved problems are the following ones:

- Which terms in the decomposition of gross operating surplus be transferred from the income accounts to the output and intermediate input accounts?
- How exactly should the reference rates be chosen?
- How exactly should the financial flows be deflated into meaningful real flows?
- What does a firm's production possibilities set look like when we take into account financing decisions?
- How exactly can asset transactions that take place within the accounting period be integrated into the analysis?

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[31] The account servicing terms involve differences between observed interest rates and reference rates and the financial intermediation terms involve differences in reference interest rates. The financial intermediation terms are approximately equal to our user cost, supplier benefit and differential risk assumption terms that appeared on the right hand side of (7).

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